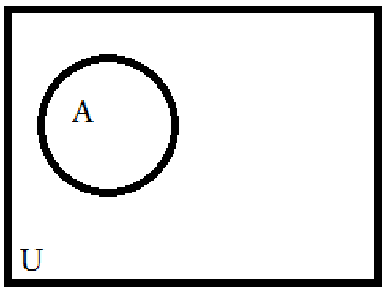
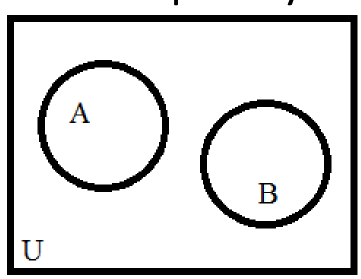
Venn Diagrams

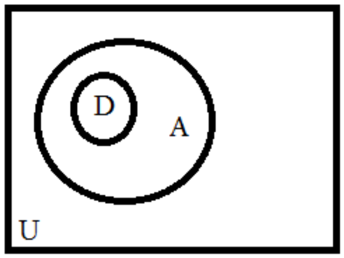
For many sets, though not all sets, it is helpful to create a picture or diagram of the relationship between the sets. A Venn Diagram does this. We start with a rectangle that represents the Universal set. 

Then we represent some set A in our diagram with a circle labeled A.



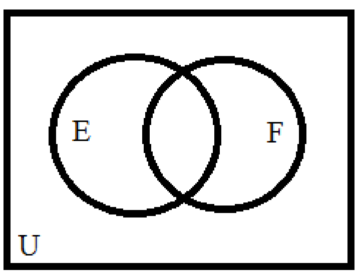
By this we mean to imply that the elements of A fill the interior of that circle. If B is another set, say it is one that is disjoint from A (remember that disjoint sets have no elements in common), then we could represent B by another circle in the diagram. Because A and B are disjoint we need to create that other circle completely outside of A. Now our diagram looks like this: 

This is a convenient spot to mention that the size of our circles is not related to the comparative cardinality of the sets A and B. The idea of the Venn Diagram is that we can readily see that the two sets are disjoint.

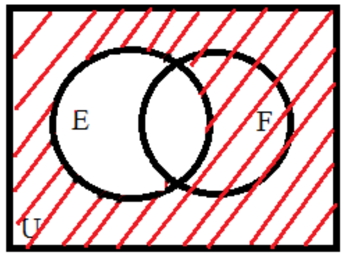
Let us look at another example. Here we will diagram A and D where D⊆A. 

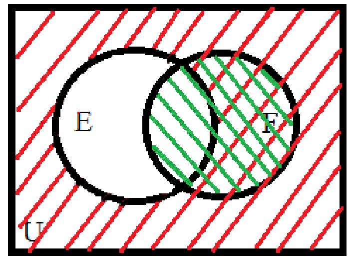
Note that D is entirely within A, which is what we would expect for a subset. As we have drawn our sets the Venn Diagram implies even more than D⊆A because there is an area inside of A but outside of D. That area must represent at least one element. As a result we infer that D⊂A.

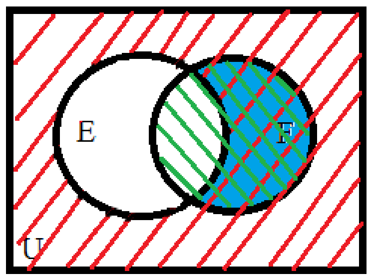
Let us try another situation. We will look at two sets E and F, where E⊈F, F⊈E, and E∩F≠∅. To diagram that relationship we see that we need two circles but they cannot be one inside the other, nor can they be separated (because they do not have a null intersection). Therefore, we need to draw this as



. Again, the size of the overlap is immaterial. We just know that there is an overlap, that something is in the intersection of the two sets.

Before moving on to use three sets we should notice that when we represent a set A by the interior of the circle, we are also representing the complement of A, i.e., A', by the region outside of the circle. Let us return to our previous diagram, but this time we will shade E^c. 

Then we could take the same diagram and shade F. 

The region that is shaded twice is $E^c \cap F$, which we will now shade as a separate region to give: 

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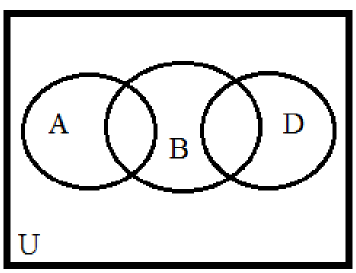
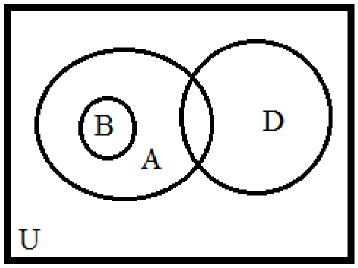
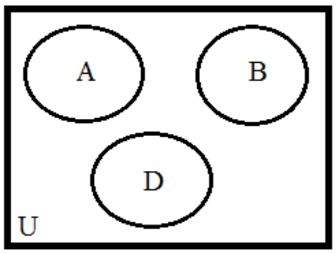
The following Venn Diagram represents sets A, B, and D:



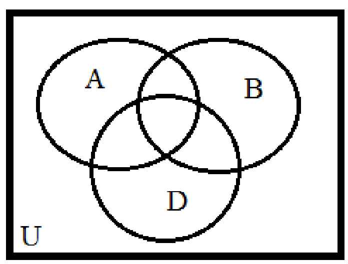
What does this diagram tell us? $A \cap B \neq \varnothing$, $D \cap B \neq \varnothing$, $A \cap D = \varnothing$, and there is at least one element of B that is neither in A nor in D.

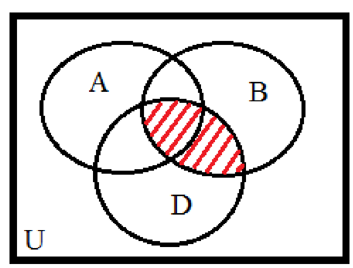
How about: 

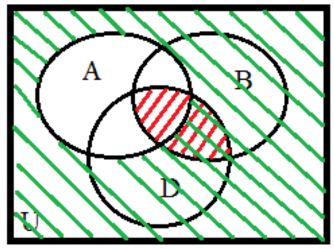
? $A \cap D \neq \emptyset$, $B \cap D = \emptyset$, $B \subset A$, and there is at least one element in A that is neither in B or D.

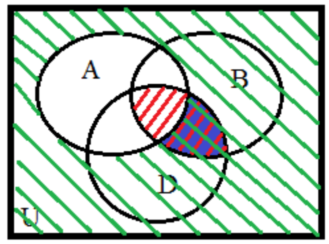
Or consider 

. For this we have $A \cap B = \emptyset$, $A \cap D = \emptyset$, $B \cap D = \emptyset$. This is a case where the three sets are mutually exclusive.

And then there is the case of 

. Here we have $A \cap B \neq \emptyset$, $A \cap D \neq \emptyset$, $B \cap D \neq \emptyset$, $A \cap B \cap D \neq \emptyset$, $(B \cap D) \cap (A^c) \neq \emptyset$, and more. For the moment, we will look at that last statement in more detail. First, we will shade $(B \cap D)$ to get 

. Then we will shade A^c to get 

. The region that is shaded twice is (B∩D) ∩ (A^c). For clarification we can shade just that region again to get 

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Venn diagrams show the relationship between sets. Circles that are separated represent disjoint sets. A circle within a circle represents a subset. Circles that overlap portions of each circle represent sets that intersect.