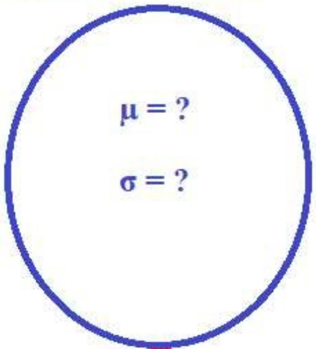
Topic 16 example 2

Hypothesis test for Population Mean, based on the sample mean, σ unknown

Start with a population with unkown mean and standard deviation

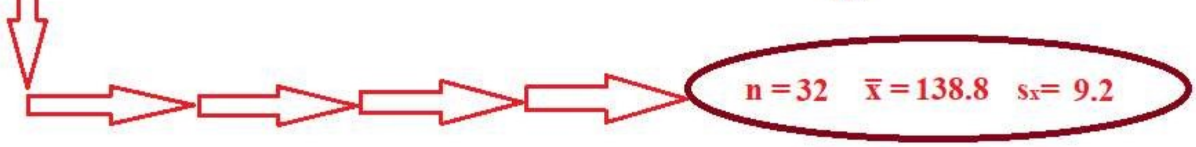


We have a null hypothesis that the mean is 135.3 and an alternative hypothesis that the mean is not equal to 135.3.

We want to test that null hypothesis at the 0.03 level of significance.

H_0: $\mu = 135.3$
H_1: $\mu \neq 135.3$
$\alpha = 0.03$

We take a random sample of size 32 and we compute the sample mean $\bar{x}$ and the sample standard deviation s_x.



Critical value approach: Because this is a two-tailed test we need to find the t value such that, for 31 degrees of freedom, P(X<-t or X>t)=0.03. We can do this via high_t <- qt(0.03/2, 31, lower.tail=FALSE) and, by symmetry of the Student's-t distribution, low_t is just the opposite value.

Now we need to translate those values back to the case where the mean is 135.3 and the standard deviation of the sample mean is $9.2/\sqrt{32}$. We do this via 135.3 + low_t*9.2/sqrt(32) and 135.3 + high_t*9.2/sqrt(32) giving a critical low value of 131.6007 and a critical high value of 138.9993.

Then, because the sample mean, 138.8, is neither less than the critical low nor greater than the critical high values we say that we do not have enough evidence to reject H_0 in favor of H_1.

Attained significance approach: We just ask how strange would it be, if H_0 is true, to get a sample mean this extreme or more extreme? To find this we first note that the sample mean is higher than the null hypothesis. Thus, we can find the probability of getting that sample mean or higher. First we "standardize" our sample mean, $\bar{x}$, via standard_t <- (138.8-135.3)/(9.2/sqrt(32)) to get the test statistic 2.152064. Then we can use the pt() function, with lower.tail=FALSE, to find the probability of getting that value or higher.
p_val <- pt(standard_t, 31, lower.tail=FALSE)
But that value needs to be doubled to account for low values that extreme or more extreme. This gives the value 0.3929236.

That value is not less than the level of significance, α=0.03, so we say that we do not have enough evidence to reject H_0 in favor of H_1.

Of course, we could just use our function, hypoth_test_unknown() to do both approaches at once.

```
#   Or we could have just used our
#       hypoth_test_unknown function to compute both
#       approaches
source("../hypo_unknown.R")
#              note the     0   to indicate that H1: mean != 135.3
hypoth_test_unknown( 135.3, 0, 0.03, 32, 138.8, 9.2)
```

```
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> #       hypoth_test_unknown function to compute both
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> source("../hypo_unknown.R")
> #              note the     0   to indicate that H1: mean != 135.3
> hypoth_test_unknown( 135.3, 0, 0.03, 32, 138.8, 9.2)
              H0_mu                  H1:           std. error 
            "135.3"       "mu !=  135.3"   "1.62634559672906" 
                  n            sig level                    t 
               "32"               "0.03"   "2.27461386471811" 
          samp mean           samp stdev            test stat 
            "138.8"                "9.2"   "2.15206411665471" 
            how far         critical low        critical high 
 "3.69930824314317"  "131.600691756857"  "138.999308243143" 
           attained             decision 
"0.0392923649996298"      "do not reject" 
```