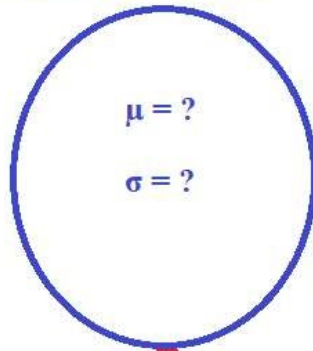


Topic 16

Hypothesis test for Population Mean, based on the sample mean, σ unknown

Start with a population with unknown mean and standard deviation



We have a **null hypothesis** that the mean is **18.6** and an **alternative hypothesis** that the mean is **less than 18.6**.

We want to test that null hypothesis at the **0.025 level of significance**.

$H_0: \mu = 18.6$
 $H_1: \mu < 18.6$
 $\alpha = 0.025$

We take a **random sample** of size **37** and we compute the sample mean \bar{x} and the sample standard deviation s_x .

$n = 37$ $\bar{x} = 17.21$ $s_x = 4.8$

Critical value approach: Find a value y such that $P(X < y) = 0.025$ for the random variable that is the mean of a sample of size 37.

To do this first find the value t such that in the Student's-t distribution with 36 degrees of freedom $P(X < t) = 0.025$. We do this via `low_t <- qt(0.025, 36)`
This gives -2.028094.

Now we need to translate that back to the case where the mean is 18.6 and the standard deviation is $4.8/\sqrt{37}$. We do this via `18.6 + low_t*4.8/sqrt(37)` to get the critical low value of 16.9996.

Then, because the sample mean, \bar{x} , is not less than the critical low we say we do not have enough evidence to reject H_0 in favor of H_1 .

Attained significance approach: We just ask how strange would it be, if H_0 is true, to get a sample mean this low or lower? To do this we first need to "standardize" our sample mean, \bar{x} . We could do this via `standard_t <- (17.21-18.6)/(4.8/sqrt(37))` and then use `pt(standard_t, 36)` to get 0.04332562.

That value is not less than the level of significance, $\alpha=0.025$, so we say we do not have enough evidence to reject H_0 in favor of H_1 .

Of course, we could just use our function, `hypo_test_unknown()` to do both approaches at once.

```
58 # Or we could have just used our
59 #   hypo_test_unknown function to compute both
60 #   approaches
61 source("../hypo_unknown.R")
62 #   note the -1 to indicate that H1: mean < 18.6
63 hypo_test_unknown( 18.6, -1, 0.025, 37, 17.21, 4.8)

> source("../hypo_unknown.R")
> #   note the -1 to indicate that H1: mean < 18.6
> hypo_test_unknown( 18.6, -1, 0.025, 37, 17.21, 4.8)
      H0_mu      H1:      std. error
      "18.6"      "mu < 18.6" "0.789115139065715"
      n      sig level      t
      "37"      "0.025"      "2.02809400098045"
samp mean      samp stdev      test stat
      "17.21"      "4.8"      "-1.76146664939886"
      how far      critical low      critical high
      "1.60039967962203"      "16.999600320378"      "n.a."
      attained      decision
      "0.0433256198786117"      "do not reject"
```