Worksheet for Two Populations, sigma unknown

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On this page we will see a number of situations with related questions. In each case, this page will give the answers to those questions. Your task is to find those same answers by inspecting the given information and/or by using a calculator or computer to produce those desired values.

Case 1:

We have a sample of size 34 from a population of unknown standard deviation. The mean of that sample is 71.04 and the sample standard deviation is 4.87. And, we have a second population, also with an unknown standard deviation. We have a sample of that population of size 53 that has a sample mean equal to 74.38 and a sample standard deviation of 3.04.

(1) What is the point estimate for the difference of the means, $\mu_1 - \mu_2$? **(Answer= -3.34)**

(2) Considering the size of our two samples and the sample standard deviations of the two samples, what is our estimate for the standard deviation of the difference of sample means? **(Answer= 0.9338)**

(3) We want a 90% confidence interval. We know that the difference of sample means is distributed as a Student's t of some degrees of freedom. If are going to take the simple approach to finding the confidence interval, what is the value of the degrees of freedom that we will use here? **(Answer= 33)**

(4) What is the **t** value that we will use so that in a Student's t distribution with our simple degrees of freedom there is 90% of the area between $^-$**t** and **t**? **(Answer= 1.6924** or **-1.6924)**

(5) Using all of those results, for this simple approach, what is the 90% confidence interval for the difference of the means? **(Answer= (-4.9203, -1.7597)**

(6) What is the value of the margin of error for that confidence interval? **(Answer= 1.5803)**

(7) If are going to take the complex approach to finding the confidence interval, what is the value of the degrees of freedom that we will use here? **(Answer= 49.5936)**

(8) What is the **t** value that we will use so that in a Student's t distribution with our complex degrees of freedom there is 90% of the area between $^-$**t** and **t**? **(Answer= 1.6762** or **-1.6762)**

(9) Using all of those results, for this complex approach, what is the 90% confidence interval for the difference of the means? **(Answer= (-4.9052, -1.7748)**

(10) What is the value of the margin of error for that confidence interval? **(Answer= 1.5652)**

Case 2:

We have a sample of size 34 from a population of unknown standard deviation. The mean of that sample is 53.92 and the sample standard deviation is 4.31. And, we have a second population, also with an unknown standard deviation. We have a sample of that population of size 60 that has a sample mean equal to 57.46 and a sample standard deviation of 2.72.

(11) What is the point estimate for the difference of the means, $\mu_1 - \mu_2$? **(Answer= -3.54)**

(12) Considering the size of our two samples and the sample standard deviations of the two samples, what is our estimate for the standard deviation of the difference of sample means? **(Answer= 0.8183)**

(13) We want a 92% confidence interval. We know that the difference of sample means is distributed as a Student's t of some degrees of freedom. If are going to take the simple approach to finding the confidence interval, what is the value of the degrees of freedom that we will use here? **(Answer= 33)**

(14) What is the **t** value that we will use so that in a Student's t distribution with our simple degrees of freedom there is 92% of the area between $^{-}$**t** and **t**? **(Answer= 1.8063** or **-1.8063)**

(15) Using all of those results, for this simple approach, what is the 92% confidence interval for the difference of the means? **(Answer= (-5.0181, -2.0619)**

(16) What is the value of the margin of error for that confidence interval? **(Answer= 1.4781)**

(17) If are going to take the complex approach to finding the confidence interval, what is the value of the degrees of freedom that we will use here? **(Answer= 48.2031)**

(18) What is the **t** value that we will use so that in a Student's t distribution with our complex degrees of freedom there is 92% of the area between $^{-}$**t** and **t**? **(Answer= 1.7884** or **-1.7884)**

(19) Using all of those results, for this complex approach, what is the 92% confidence interval for the difference of the means? **(Answer= (-5.0035, -2.0765)**

(20) What is the value of the margin of error for that confidence interval? **(Answer= 1.4635)**

Case 3:

We have a sample of size 55 from a population of unknown standard deviation. The mean of that sample is 68.55 and the sample standard deviation is 11.46. And, we have a second population, also with an unknown standard deviation. We have a sample of that population of size 53 that has a sample mean equal to 66.28 and a sample standard deviation of 10.01.

(21) What is the point estimate for the difference of the means, $\mu_1 - \mu_2$? (**Answer= 2.27)**

(22) Considering the size of our two samples and the sample standard deviations of the two samples, what is our estimate for the standard deviation of the difference of sample means? **(Answer= 2.0684)**

(23) We want a 98% confidence interval. We know that the difference of sample means is distributed as a Student's t of some degrees of freedom. If are going to take the simple approach to finding the confidence interval, what is the value of the degrees of freedom that we will use here? **(Answer= 52)**

(24) What is the **t** value that we will use so that in a Student's t distribution with our simple degrees of freedom there is 98% of the area between $^{-}$**t** and **t**? **(Answer= 2.4002** or **-2.4002)**

(25) Using all of those results, for this simple approach, what is the 98% confidence interval for the difference of the means? **(Answer= (-2.6947, 7.2347)**

(26) What is the value of the margin of error for that confidence interval? **(Answer= 4.9647)**

(27) If are going to take the complex approach to finding the confidence interval, what is the value of the degrees of freedom that we will use here? **(Answer= 105.0043)**

(28) What is the **t** value that we will use so that in a Student's t distribution with our complex degrees of freedom there is 98% of the area between $^{-}$**t** and **t**? **(Answer= 2.3624** or **-2.3624)**

(29) Using all of those results, for this complex approach, what is the 98% confidence interval for the difference of the means? **(Answer= (-2.6164, 7.1564)**

(30) What is the value of the margin of error for that confidence interval? **(Answer= 4.8864)**

Case 4:

We have two populations. We do not know their standard deviation. We draw a random sample from each population. Here is the first:

Table 1: Sample from Population 4.1																	
216.1	213.7	208.2	212.7	214.5	219.2	212.4	215.0	216.1	214.8	198.9	202.9	198.6	219.7	204.5	220.8	216.0	206.7
220.9	207.8	209.5	213.7	216.1	197.6	218.3	205.9	199.9	218.9	215.0	198.8	214.1	208.6	219.4	225.2	216.2	211.0
211.4	211.1	212.1	207.2	218.5	215.2	206.1	200.5	209.8	216.0								

You can generate this set of data using the command **gnrnd4(1690604504,7902117).**

Then we draw a random sample from the second population. Here is that sample:

Table 2: Sample from Population 4.2																			
220.3	220.4	211.7	210.6	221.4	220.1	224.0	210.9	218.7	218.8	214.5	225.8	230.3	219.5	217.7	218.0	223.5	216.4	221.8	221.1
221.9	219.9	222.2	223.3	220.2	213.7	208.7	217.0	211.3	222.2	205.7	220.6	221.1	217.6	217.7	216.5	219.5	211.1	218.0	216.7
208.5																			

You can generate this set of data using the command **gnrnd4(1523564004,5502173).**

(31) What is the sample size of the first sample? **(Answer= 46)**

(32) What is the sample size of the second sample? **(Answer= 41)**

(33) What is the sample mean of the first sample? **(Answer= 211.6435)**

(34) What is the sample mean of the second sample? **(Answer= 218.0220)**

(35) What is the sample standard deviation of the first sample? **(Answer= 6.8382)**

(36) What is the sample standard deviation of the second sample? **(Answer= 5.1050)**

(37) What is the point estimate for the difference of the means, $\mu_1 - \mu_2$? (**Answer=-6.3785)**

(38) Considering the size of our two samples and the standard deviations of the two samples, what is our estimate for the standard deviation of the difference of sample means? **(Answer= 1.2854)**

(39) We want a 95.5% confidence interval. We know that the difference of sample means has a Student's t distribution with some number of degrees of freedom. If we want to take the simple approach to finding the confidence interval then what is the value of the degrees of freedom that we will use? **(Answer= 40)**

(40) What is the **t** value that we will use so that in a Student's t distribution, using the simple degrees of freedom, there is 95.5% of the area between $^-$**t** and **t**? **(Answer= 2.0695** or **-2.0695)**

(41) Using all of those results, what is the 95.5% confidence interval, using the simple approach, for the difference of the means? **(Answer= (-9.0386, -3.7184)**

(42) What is the value of the margin of error for that confidence interval? **(Answer= 2.6601)**

(43) If we want to take the complex approach to finding the confidence interval then what is the value of the degrees of freedom that we will use? **(Answer= 82.5567)**

(44) What is the **t** value that we will use so that in a Student's t distribution, using the complex degrees of freedom, there is 95.5% of the area between $^-$**t** and **t**? **(Answer= 2.0356** or **-2.0356)**

(45) Using all of those results, what is the 95.5% confidence interval, using the complex approach, for the difference of the means?

(Answer= (-8.9950, -3.7620)

(46) What is the value of the margin of error for that confidence interval? **(Answer= 2.6165)**

Case 5:

We have two populations. We do not know their standard deviation. We draw a random sample from each population. Here is the first:

Table 1: Sample from Population 4.1																	
-65.6	-53.7	-54.5	-56.1	-56.4	-68.2	-56.5	-79.5	-63.3	-59.8	-55.6	-61.8	-64.8	-62.7	-76.9	-58.9	-69.0	-36.0
-73.9	-74.1	-84.8	-67.8	-58.6	-45.2	-63.7	-35.1	-50.0	-37.9	-63.5	-50.3	-61.4	-76.1	-81.3	-70.9	-49.2	-81.5
-55.1	-74.5	-59.4	-69.3	-64.7	-64.5	-57.1	-58.7	-65.1	-61.4	-72.4	-38.6	-75.4	-63.4	-59.2	-47.9	-46.2	-64.2
-65.0	-77.1	-60.8	-84.9	-57.6	-73.8	-63.5	-56.9	-68.4	-50.2	-36.9	-66.1	-75.3	-63.9	-66.0	-60.4	-62.7	-63.3
-49.6	-56.8																

You can generate this set of data using the command **gnrnd4(6744157304,10200633).**

Then we draw a random sample from the second population. Here is that sample:

Table 2: Sample from Population 4.2																		
-82.6	-62.1	-71.7	-58.1	-78.3	-78.0	-68.3	-68.0	-74.2	-80.6	-70.6	-61.4	-85.8	-83.8	-64.2	-81.9	-57.5	-72.8	-61.3	-76.4
-70.0	-65.1	-72.9	-81.2	-67.2	-71.6	-70.4	-73.5	-78.1	-62.7	-68.2	-70.3	-58.7	-75.0	-69.7	-68.2	-67.1	-66.7	-76.8	-72.1

You can generate this set of data using the command **gnrnd4(6746183904,7200704).**

(47) What is the sample size of the first sample? **(Answer= 74)**

(48) What is the sample size of the second sample? **(Answer= 40)**

(49) What is the sample mean of the first sample? **(Answer= -61.9041)**

(50) What is the sample mean of the second sample? **(Answer= -71.0775)**

(51) What is the sample standard deviation of the first sample? **(Answer= 11.2817)**

(52) What is the sample standard deviation of the second sample? **(Answer= 7.3175)**

(53) What is the point estimate for the difference of the means, $\mu_1 - \mu_2$? (**Answer=9.1734)**

(54) Considering the size of our two samples and the standard deviations of the two samples, what is our estimate for the standard deviation of the difference of sample means? **(Answer= 1.7489)**

(55) We want a 83.5% confidence interval. We know that the difference of sample means has a Student's t distribution with some number of degrees of freedom. If we want to take the simple approach to finding the confidence interval then what is the value of the degrees of freedom that we will use? **(Answer= 39)**

(56) What is the **t** value that we will use so that in a Student's t distribution, using the simple degrees of freedom, there is 83.5% of the area between $^{-}$**t** and **t**? **(Answer= 1.4150** or **-1.4150)**

(57) Using all of those results, what is the 83.5% confidence interval, using the simple approach, for the difference of the means? **(Answer= (6.6988, 11.6481)**

(58) What is the value of the margin of error for that confidence interval? **(Answer= 2.4747)**

(59) If we want to take the complex approach to finding the confidence interval then what is the value of the degrees of freedom that we will use? **(Answer= 108.1860)**

(60) What is the **t** value that we will use so that in a Student's t distribution, using the complex degrees of freedom, there is 83.5% of the area between $^{-}$**t** and **t**? **(Answer= 1.3979** or **-1.3979)**

(61) Using all of those results, what is the 83.5% confidence interval, using the complex approach, for the difference of the means? **(Answer= (6.7287, 11.6182)**

(62) What is the value of the margin of error for that confidence interval? **(Answer= 2.4448)**

Case 6:

We have two populations, neither of which has a known standard deviation. We want test the null hypothesis that the means of the two populations are the same versus the alternative hypothesis that mean of the first population is not equal to the mean of the second population. We will do this test at the 0.04 level of significance. We draw samples from each population. The sample from the first population has size 62 and sample mean equal to 72.47 and sample standard deviation equal to 6.98. The sample from the second population has size 30 and sample mean equal to 69.61 and sample standard deviation equal to 5.04.

(63) State the null hypothesis. **(Answer= H_0: $\mu_1=\mu_2$)**

(64) State the alternative hypothesis. **(Answer= H_1: $\mu_1 \neq \mu_2$)**

(65) Give the value of the difference of the two means $\left(\overline{x}_1 - \overline{x}_2\right)$. **(Answer= 2.8600)**

(66) Give the value of the standard deviation of the difference of the two means. **(Answer= 1.2777)**

(67) Give the t-value that we would use [diff of means/estimate of sd] to test the alternative hypothesis. **(Answer= 2.2384)**

(68) For the **simple approach** to this problem, give the number of degrees of freedom that we will use? **(Answer= 29)**

(69) Give the critical value (or values) using this simple approach. **(Answer= low is -2.747 and high is 2.747)**

(70) Give the attained (achieved) significance for the difference of the two means, using the simple approach. **(Answer= 0.0330)**

(71) Therefore, using the simple approach, do we reject or not reject H_0? **(Answer= Reject)**

(72) If we want to take the **complex approach** to teesting the hypothesis then what is the value of the degrees of freedom that we will use? **(Answer= 76.4865)**

(73) Give the critical value (or values) using this complex approach. **(Answer= low is -2.670 and high is 2.670)**

(74) Give the attained (achieved) significance for the difference of the two means, using the complex approach. **(Answer= 0.0281)**

(75) Therefore, using the complex approach, do we reject or not reject H_0? **(Answer= Reject)**

Case 7:

We have two populations, neither of which has a known standard deviation. We want test the null hypothesis that the means of the two populations are the same versus the alternative hypothesis that mean of the first population is not equal to the mean of the second population. We will do this test at the 0.06 level of significance. We draw samples from each population. The sample from the first population has size 65 and sample mean equal to 66.92 and sample standard deviation equal to 6.86. The sample from the second population has size 83 and sample mean equal to 68.72 and sample standard deviation equal to 4.95.

(76) State the null hypothesis. **(Answer= H_0: $\mu_1=\mu_2$)**

(77) State the alternative hypothesis. **(Answer= H_1: $\mu_1 \neq \mu_2$)**

(78) Give the value of the difference of the two means $\left(\overline{x}_1 - \overline{x}_2\right)$. **(Answer= -1.8000)**

(79) Give the value of the standard deviation of the difference of the two means. **(Answer= 1.0096)**

(80) Give the t-value that we would use [diff of means/estimate of sd] to test the alternative hypothesis. **(Answer= -1.7830)**

(81) For the **simple approach** to this problem, give the number of degrees of freedom that we will use? **(Answer= 64)**

(82) Give the critical value (or values) using this simple approach. **(Answer= low is -1.933 and high is 1.933)**

(83) Give the attained (achieved) significance for the difference of the two means, using the simple approach. **(Answer= 0.0793)**

(84) Therefore, using the simple approach, do we reject or not reject H_0? **(Answer= Do Not Reject)**

(85) If we want to take the **complex approach** to teesting the hypothesis then what is the value of the degrees of freedom that we will use? **(Answer= 112.2650)**

(86) Give the critical value (or values) using this complex approach. **(Answer= low is -1.918 and high is 1.918)**

(87) Give the attained (achieved) significance for the difference of the two means, using the complex approach. **(Answer= 0.0773)**

(88) Therefore, using the complex approach, do we reject or not reject H_0? **(Answer= Do Not Reject)**

Case 8:

We have two populations, neither of which has a known standard deviation. We want test the null hypothesis that the means of the two populations are the same versus the alternative hypothesis that mean of the first population is less than the mean of the second population. We will do this test at the 0.02 level of significance. We draw samples from each population. The sample from the first population has size 58 and sample mean equal to 113.65 and sample standard deviation equal to 5.50. The sample from the second population has size 41 and sample mean equal to 115.73 and sample standard deviation equal to 3.77.

(89) State the null hypothesis. **(Answer= H_0: $\mu_1=\mu_2$)**

(90) State the alternative hypothesis. **(Answer= H_1: $\mu_1 < \mu_2$)**

(91) Give the value of the difference of the two means $\left(\overline{x}_1 - \overline{x}_2\right)$. **(Answer= -2.0800)**

(92) Give the value of the standard deviation of the difference of the two means. **(Answer= 0.9318)**

(93) Give the t-value that we would use [diff of means/estimate of sd] to test the alternative hypothesis. **(Answer= -2.2323)**

(94) For the **simple approach** to this problem, give the number of degrees of freedom that we will use? **(Answer= 40)**

(95) Give the critical value (or values) using this simple approach. **(Answer= low is -1.978)**

(96) Give the attained (achieved) significance for the difference of the two means, using the simple approach. **(Answer= 0.0156)**

(97) Therefore, using the simple approach, do we reject or not reject H_0? **(Answer= Reject)**

(98) If we want to take the **complex approach** to teesting the hypothesis then what is the value of the degrees of freedom that we will use? **(Answer= 96.9314)**

(99) Give the critical value (or values) using this complex approach. **(Answer= low is -1.940)**

(100) Give the attained (achieved) significance for the difference of the two means, using the complex approach. **(Answer= 0.0139)**

(101) Therefore, using the complex approach, do we reject or not reject H_0? **(Answer= Reject)**

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